The Improved GP 2 Compiler

11th International Workshop on Graph Computation Models

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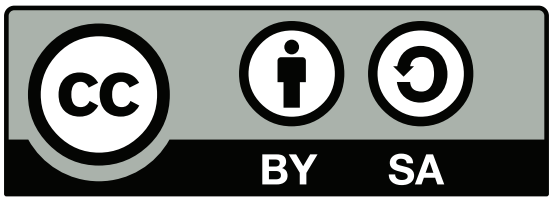

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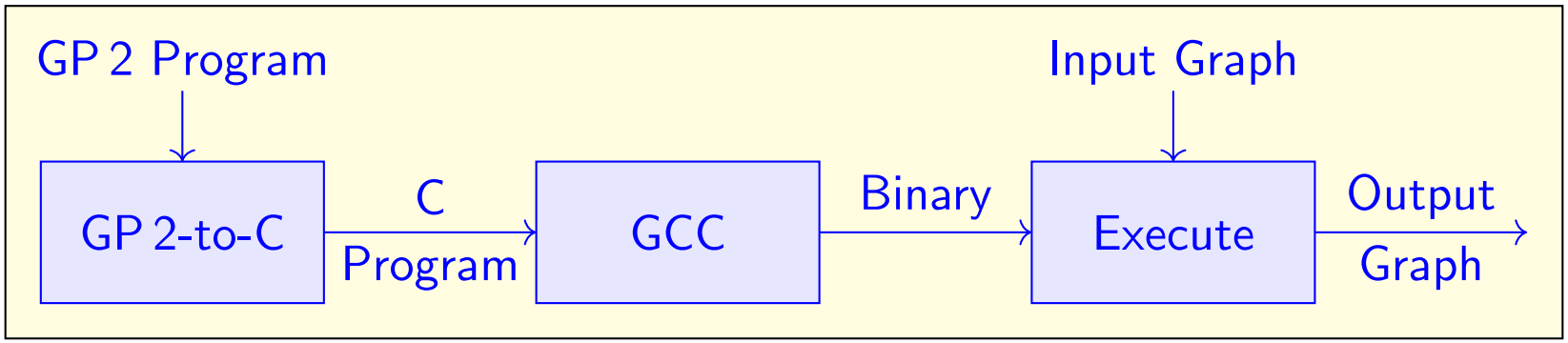

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- Roots in rules must match roots in the host graph so that matches are restricted to the neighbourhood of the host graph's roots.
- Using rooted rules, some graph algorithms can be implemented to run in linear time on graphs of bounded degree: computing a 2-colouring (Bak and Plump 2016); topological sorting of acyclic graphs (Campbell, Courtehoute, and Plump 2019). MSTs can be computed in linearithmic time (Courtehoute and Plump 2020).

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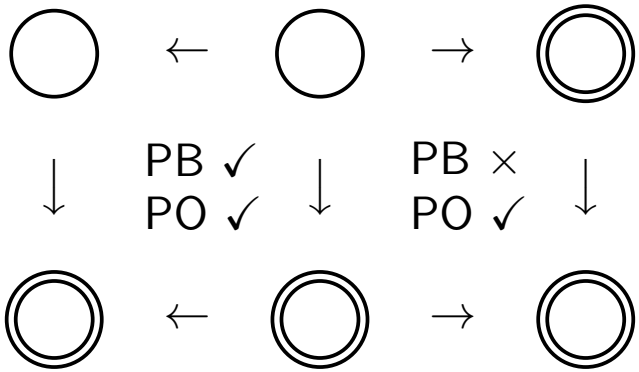
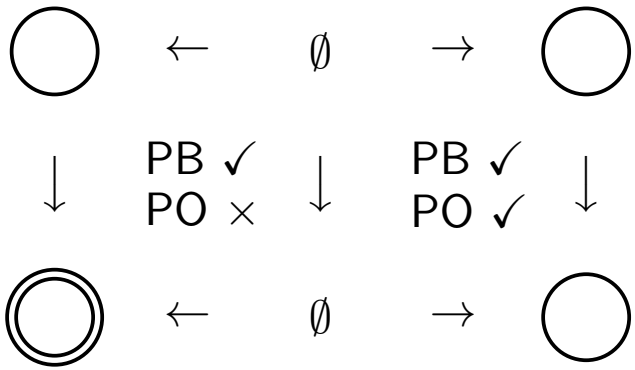


(a) Not invertible



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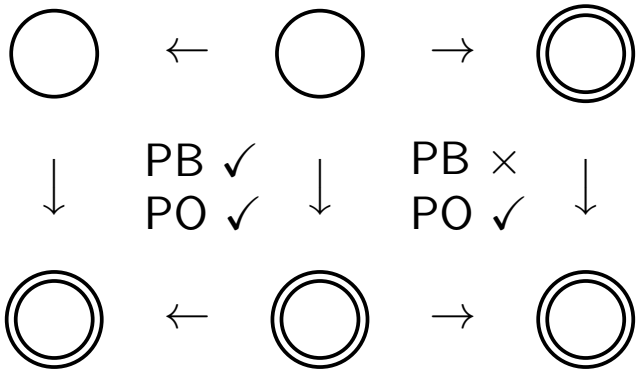

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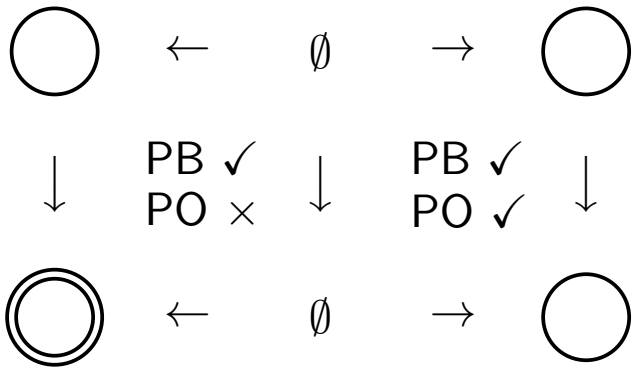

(b) Not DPO rewriting

The 2nd example is due to Plump and Wulandari (unpublished). These problems were fixed by Campbell (2019) by requiring root-reflecting morphisms for rooted graph transformation with relabelling.

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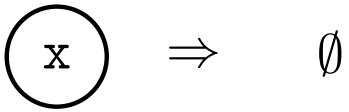


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node(x:list)
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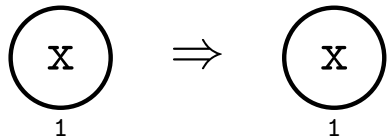

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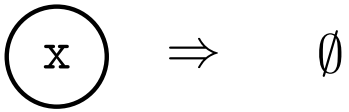


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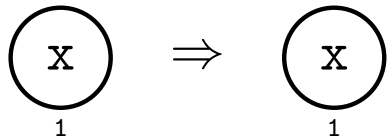

The above program evaluates to `fail` if and only if the input graph is discrete (has no edges). We would expect it to run in linear time, but it actually takes quadratic time on discrete graphs!

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Root nodes are tracked by a linked list, each entry holding a pointer to a root node. Iterating through or deleting root nodes takes constant time, if there is an upper bound on the number of root nodes.

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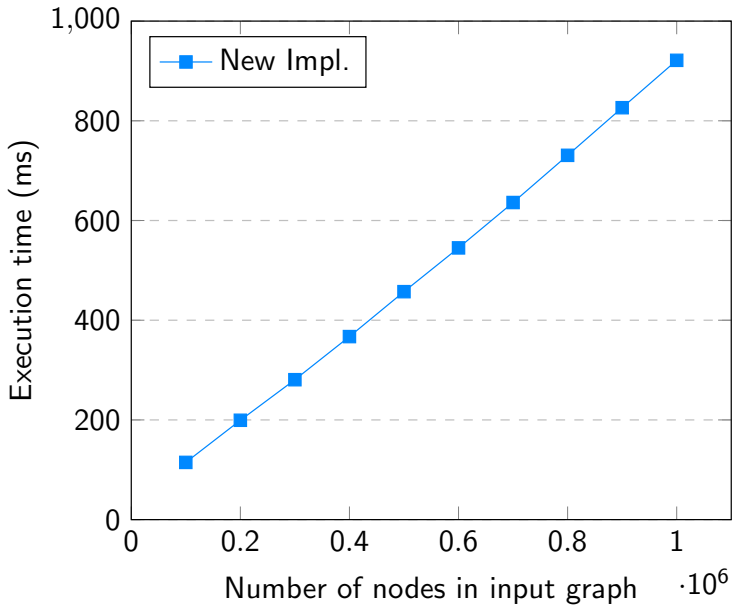
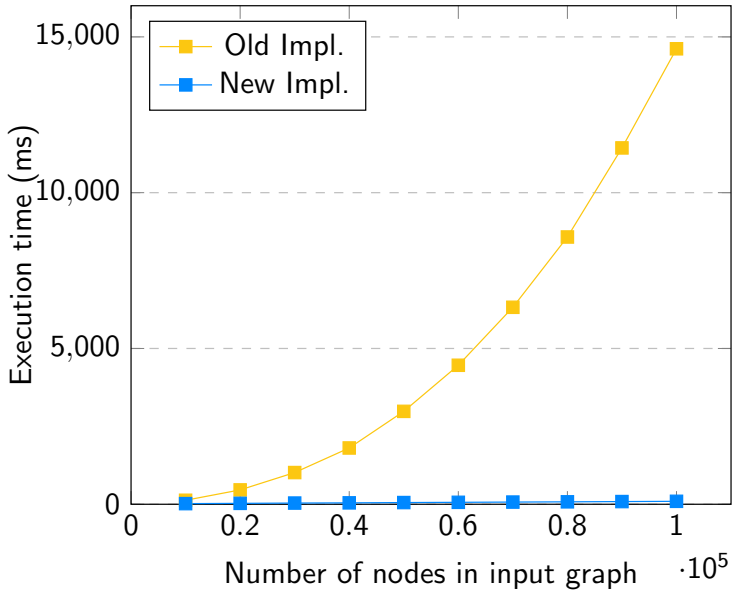
There were lots of other internal changes to enable this, such as the replacement of integer IDs by pointers, and re-implementation of edge lists which leads to good performance of our next examples.

Discrete Graph Testing Timing





Binary DAG Testing Program

```
Main = (init; Reduce!; if flag then break)!; if flag then fail
Reduce = up!; try Delete else set_flag
Delete = {del1, del1_d, del21, del21_d, del22, del22_d, del0}
```

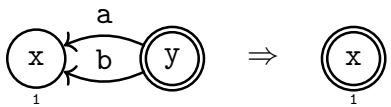
init(x:list)

up(a,x,y:list)

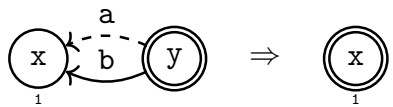
del1(a,x,y:list)

del1_d(a,x,y:list)

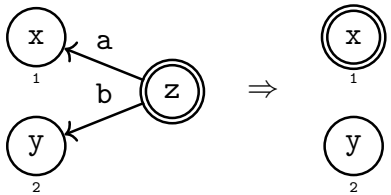
del21(a,b,x,y:list)



del21_d(a,b,x,y:list)



del22(a,b,x,y,z:list)



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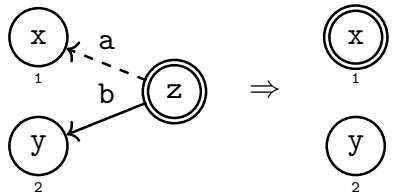
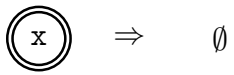


del0(x:list)



set_flag(x:list)

flag(x:list)

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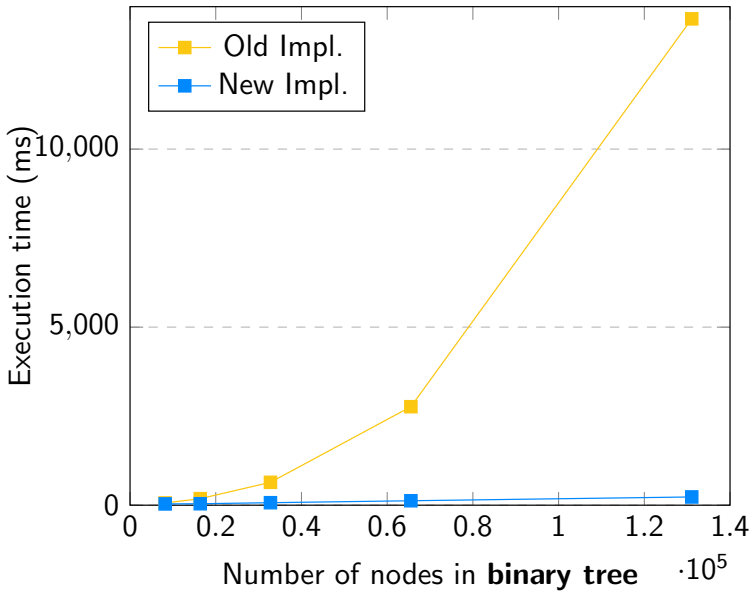
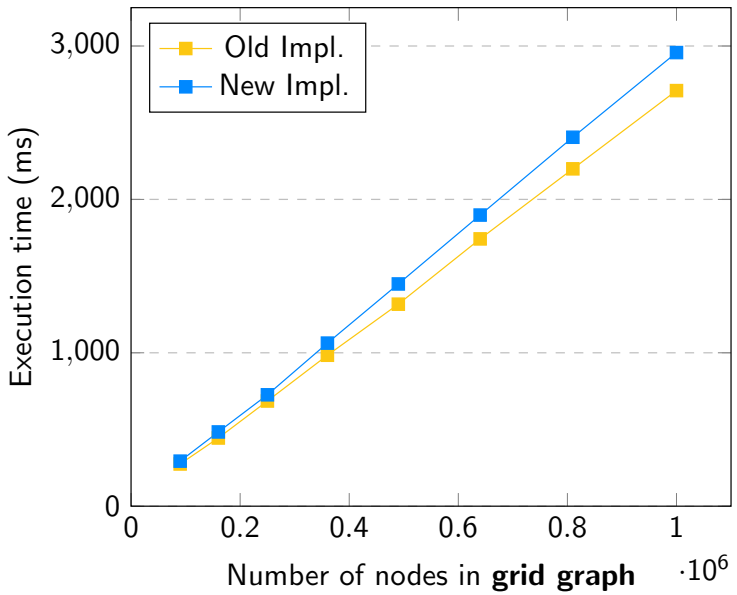
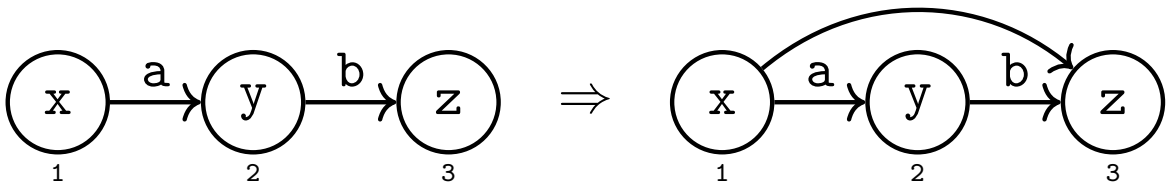




Transitive Closure Program

```
Main = link!

  link(a,b,x,y,z:list)
```

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  where not edge(1,3)
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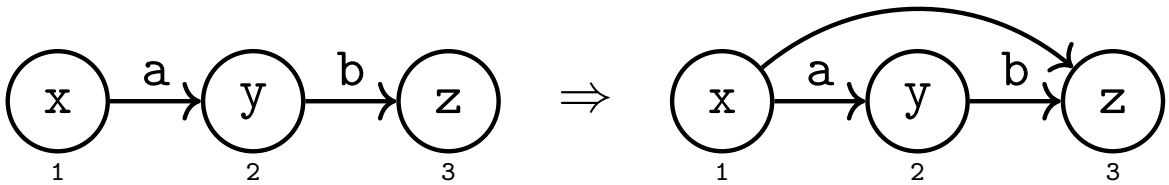


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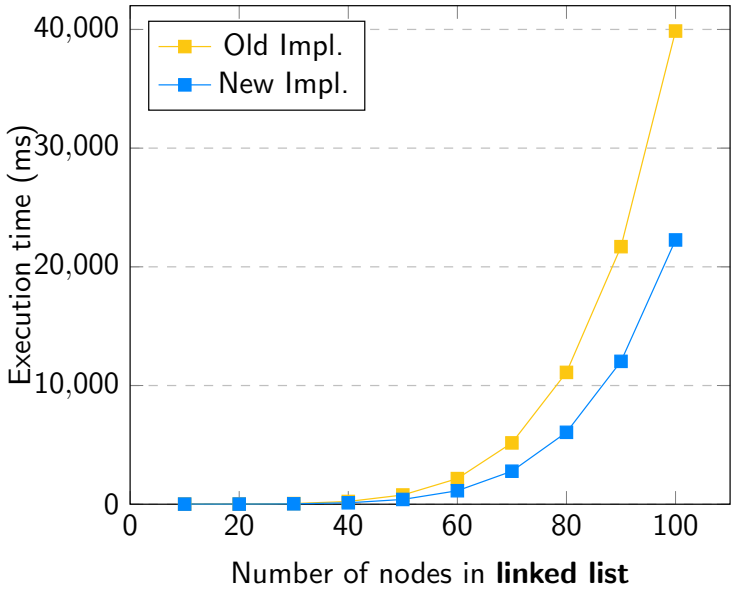

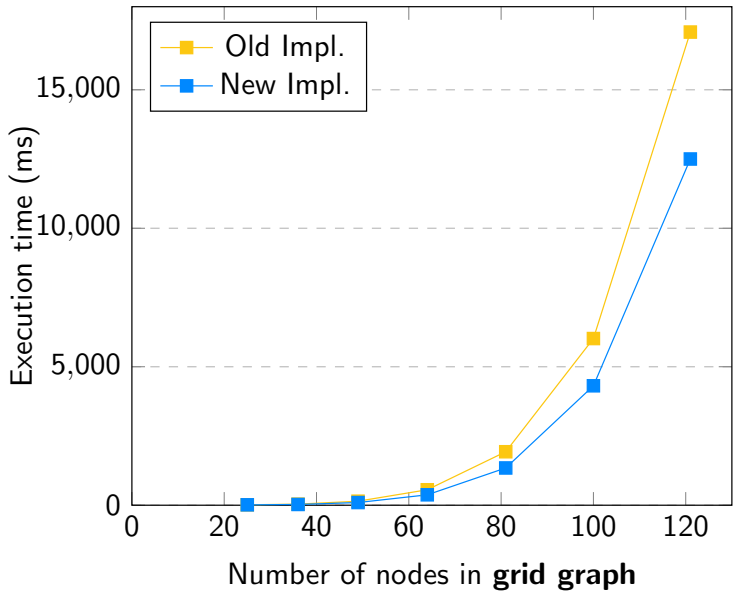

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https://github.com/UoYCS-plasma/GP2

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